Astronomy & Astrophysics manuscript no. comov
November 7, 2005

Comoving frames and active longitudes: Does the Sun have face?

J. Pelt[1], J. Brooke[2,3], M. Korpi[4], and I. Tuominen[4]

[1] Tartu Observatory, 61602 Tõravere, Estonia
[2] Manchester Computing, University of Manchester, Oxford Road, Manchester, M13 9PL, UK
[3] Department of Mathematics, University of Manchester, Oxford Road, Manchester, M13 9PL, UK
[4] Helsinki, Finland

Received —; accepted —

ABSTRACT

Context. Using Greenwich sunspot data for 120 years it was recently observed that activity regions on the Sun's surface tend to lie along smoothly changing longitude strips 180°apart from each other. After critique of the original paper, the new arguments were found to substantiate the claim.
Aims. In this paper we analyse new arguments in some detail.
Methods. To check basic arguments about persistent longitudes we tried to reproduce results obtained in previous papers. Our computations and additional cross-checks revealed number of inconsistencies.
Results. As a result of analysis we can say that evidence for essential and persistent non-axisymmetricity in sunspot distibution is still week.

Key words. Sun: activity – Sun: magnetic fields – sunspots – methods: statistical

1. Introduction

In a recent row of papers by Berdygina & Usoskin 2003 (hence BU), Usoskin, Berdyugina & Poutanen 2005 (UBP) and finally Berdygina et al. 2005 (BMSU) authors promote an idea about persistent (during 120 years) active longitude belts which are separated by 180° and which drift smoothly along longitudes. As a response to the first paper Pelt et al. 2005 demonstrated that large part (or all) of the effect can be result of particular data treatment. Essentially, it was shown that diagrams similar to these, used as a proof in BU, can be obtained even from random data. Two new papers UBP and BMSU claim that they can now confirm previous results using much more simpler constructions which do not use data processing steps criticized.

Before going to the new arguments we must stress from the very beginning that even when the new results in UBP and BMSU are reliable, they do not confirm spot migration model which was given in the first paper (BU). And this is simply beacuse the moving along phases frame in the first paper did about 28 extra rotations if to compare with Carrington frame, contrasted with the new values of about ≈ 11 rotations for Northern hemisphere and about ≈ 14 for Southern hemisphere. Consequently models and corresponding results in the two new papers must be looked upon as a totally new development. It is interesting that authors do not mention this in their new papers. *Vice versa* - they even refer to particular results (for instance period of the flip-flop phenomena - 3.7 years) which is computed from old (and then wrong) migration frame. Reader can convince himself just by comparing Fig. 1 in (UBP) where migration frame for years 1954-1965 is given with similar frame in lower left part of Fig. 3 of BU.

Below we will analyse comoving frames presented in UBP and BMSU in three steps. First we will show, using extremely simple statistical arguments, that comoving frames, which help to clarify basic properties of the sunspot longitudinal distribution, indeed exist and can be useful.

Then we report how we (unsuccessfuly) tried to recover optimal migration frames kind of these in second paper (UBP) and speculate about possible reasons of our failure.

Finally we comment on significance estimation procedures used in UBP and by us. From this analysis follows that beyond reasonable doubt all solutions where bimodal distributions of longitudes show up are just statistical fluctuations.

Send offprint requests to: J. Pelt

1.1. Data and conventions

Our input data for statistical analysis was downloaded from the same Science at NASA web site [1] as in BU and UBP. The provided in site format description of the data sets occured to be incomplete and to unscramble data we used original document from NOAA Satellite and Information Service Site [2]. During the data checking we found some outliers in the data. The record from observations of year 1980 which referred to longitude 408.9° was skipped and for couple of records where longitude was only slightly more than 360° we subtracted 360° from given value (assuming circularity). The subset of full data set for years 1878-1996 was then singled out to be compatible with papers under scrutiny. In the terms of Carrington rotations we used rotations 325-1917 for both hemispheres. For each particular sunspot group or single spot we selected only record of its first occurence as was done in original papers. It is important to note that some sunspot group numbers in data base are mutiply used. To count them as separate entities we used 21-th column in records (this was just the piece of information which was missing in the first format description mentioned above). Finally we had two data sets: 1593 rotations and 18680 records for Northern hemisphere and 1593 rotations and 17966 records for Southern hemisphere, data sets are available on the web [3]. In original papers "about 1600" rotations was used and "about 40000" records.

To simplify discussions below we will often use instead Carrington rotation numbers ordinal numbers in our data set (hence $1, \ldots, 1593$). We also use in all plots and formulas basic phase interval $0 - 360$. As a result, some formulas can differ from these in original papers. However, the original forms can easily be recovered by using apropriate phase shift.

2. Comoving frames

The two papers under scrutiny and arguments therein depend strongly on assumption that first occurences of single sunspots and sunspot groups (or their density waves) drift along longitudes smoothly, according to certain law, whose exact form is to be established. To analyse such systematic drifts we will use following scheme. The full set of sunspot records is divided into set of subintervals of length 27.2753 days, so called Carrington rotations. For each particular rotation $i, i = 1, \ldots, N$ the phase shift Ω_i is defined (in degrees per day). For the full time span the shifts accumulate according to the formula

$$\Lambda_i = \Lambda_0 + T_C \sum_{j=1}^{i} (\Omega_j - \Omega_C), \tag{1}$$

where $T_C = 25.38$, $\Omega_C = 360^0/T_C$ and Λ_0 is general phase shift to be computed later. These cumulative sums Λ_i (in degrees) are used as a flow model to describe drift of sunspots. When we substract the flow model from actual longitudes

$$\tilde{\lambda}_{ki} = \lambda_{ki} - \Lambda_i - m \times 360, \tag{2}$$

and select m so that results will lay in interval $[0, 360]$ we will get certain distribution of longitudes in so called *rest frame*. The flow model itself is then *comoving frame*, its rotation phase is determined by Eq. 1.

Our flow model is very similar to that used in UBP, the only difference is usage of $\Omega_j - \Omega_C$ instead of $\Omega_C - \Omega_j$ in UBP. As seen from Eq. 1 in our case for values of $\Omega_i > \Omega_C$ model phases are pushed forward, otherwise phases are pulled back.

Particular drift models can depend on a set of parameters. To find best model we define certain merit function (criterion) for phases in rest frame and then seek parameter combination which corresponds to extremum value of merit function. For uniformity we will use functions whose *minima* are indicators of seeked for "good" phase distributions.

2.1. Best frames

Let us now demonstrate that the formalism just described is sound and useable. We start from the simplest system of shifts $\Omega_i = \Omega_0$, that is, we assign for each Carrington rotation constant shift Ω_0. To define our first merit function we divide all Carrington rotations into ten piece groups and all longitudes (phases) into 6°wide cells. We work with all together 1593 separate rotations and correspondingly we have here a matrix with 60×160 cells. For each set of free parameters (here Λ_0 and Ω_0) we compute merit function in a following simple way: each of the corrected phases $\tilde{\lambda}_{ki}$ belongs to certain cell; some of the cells remain empty because there is not phases in particular rotation and cell of phases; as a criterion we use ratio of occupied cells to total number of phases.

The rationale of this scheme is obvious. If the spots or spot groups have persistent longitudes (in a corresponding frame) then they tend to stay (at least statistically so) in one and the same cell. By fixing cell length in rotations, we also fix for how long time we assume the activity to be persistent. The width of the cells fixes the amount of allowed phase dispersion during the number of rotations.

It occurs that this, nearly trivial scheme, to seek best comoving frames, works unexpectedly well. On Fig. 1 we depict how our merit function changes with changing of the input parameter Ω_0 in a wide range of values. The curve is somewhat noisy, but still demonstrates well how the phases start to line up for Ω_0 values larger than $\approx$ 14.1 deg/day and then how the scatter is again increasing starting from approximately $\approx$ 14.6 degrees per day. The wide depression is too fluctuating to single out one – best – frame for all latitudes together.

The generalization which takes into account differential rotation is obvious. We just need to preselect certain

[1] http://science.msfc.nasa.gov/ssl/pad/solar/greenwch.htm
[2] ftp://ftp.ngdc.noaa.gov/STP/SOLAR DATA/SUNSPOT REGIONS/GREENWICH/GROUPS/format.grp
[3] http://www.aai.ee/ pelt/soft.htm

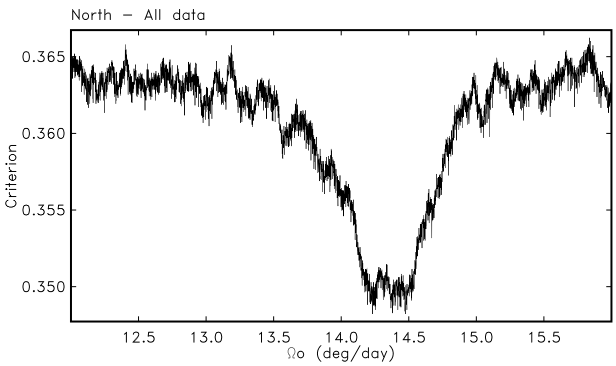

Fig. 1. Merit function plotted against Ω_0. The best mean corotating frame can be singled out by selecting parameter combination which minimizes the number of occupied longitude cells in 60×160 matrix. The dependency on Λ_0 is "minimized out". Differential rotation is not taken into account and therefore we see only wide depression.

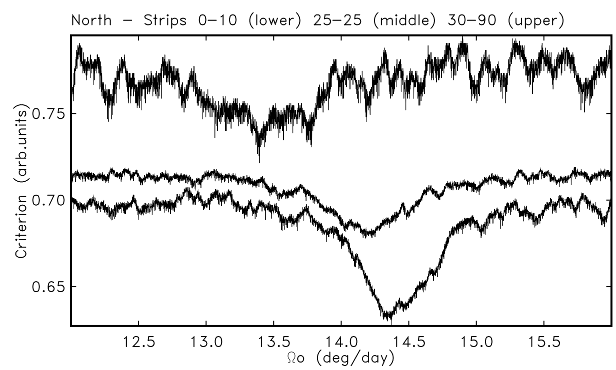

Fig. 2. Merit functions for the subsets of data in different latitude strips. The minima are now shifted and this is a result of differential rotation. By computing such "spectra" for diffrent latitude strips we can build a full rotation profile. Speed for equatorial rotation is ≈ 14.37, for middle lattitudes ≈ 14.18 and for high latitudes ≈ 13.40 degrees per day.

strips in latitudes and do our analysis for these subsets. And indeed, as seen from Fig. 2, the method works here quite well. We can see how the principal minima are now shifted. The values of Ω_0 for different strips can be used to build latitude dependent rotation profile. The criterion curves are somewhat noisy, but this can be improved by smoothing the curves.

The chosen parameters for cell building are nothing special, the obtained results are quite robust. This is not major topic of this paper and we leave detailed analysis of this trivial method to be published later. In current context the important conclusions are:

– The idea about comoving frames (at least with fixed speed) is useful even when applied to the full 120 year long data set.
– The scheme with phase corrections is statistically sound and robust procedure which helps to compute optimal parameters for comoving frame.
– The comoving frames in different latitude strips have quite stable rotation speeds which show itself as a principal merit function minima.
– The minima of the merit function are located inside wide depressions and that shows that they are not minor fluctuation like peaks.

We also note that method does not use sunspot areas at all - all the information about sunspot rotations are extracted from longitudes and corresponding time points.

2.2. Frames with changing speed

Instead of analysing latitude strips separately we can try to build one frame for all data with changing rotation speed. Following the UBP we will now include for each rotation an additional term:

$$\Omega_i = \Omega_0 - B \sin^2 \langle \psi \rangle_i, \qquad (3)$$

where $\langle \psi_i \rangle_i$ is a area weighted average latitude for i-th Carrington rotation. Symbol B is a new parameter which measures the amplitude of latitude dependent part. As we know from ubiquitus "butterfly diagram' the mean latitude of activity indicators drifts during solar cycle. For higher latitudes the differential rotation speed is slower and correspondingly our comoving frame will slow down. If majority of spots rotate near the equator then our frame is speeded up. In middle latitudes it is at rest.

To get best triple of frame parameters we need to compute merit function for large grid of trials. The best frame for parameter grid $[13.5 - 15.0; 0.001] \times [0.0 - 6.0; 0.02] \times [1 - 5; 1]^4$ is depicted on Fig. 3 and corresponding shifts for comoving frame on Fig. 4. (We need to evaluate merit only for six different degrees because there is only six different start positions for 6° cells.)

It is important to stress that strongest minimum for variable speed frame is significantly deeper if to compare with fixed rate frame. Consequently it indeed models real features in sunspot longitude distribution. We are not saying here that cumulative sums of Ω_i-s is best conceivable model. It is just one possible solution and gives reasonably good results. Probably it can be improved if we take into account that in the solar minimum times the average latitudes are formed from spots of the old cycle and new cycle all together (see Pelt 2000 how the different cycles can be separated).

3. Active longitudes

The basic claim of the BU,UBP and BMSU papers is that for the last 120 years there can be seen in sunspot lon-

[4] Here and below we use systematic notation for computation grids. Inside squared brackets we give first minimum and maximum values for parameter followed by stepsize. Different parameters can be distinguished from context.

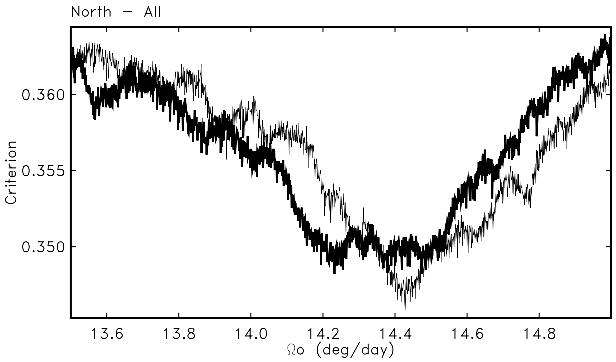

Fig. 3. Merit functions computed for all data. When $B = 0$ (fixed rate frame, thick line) the minimum is rather wide, and it is quite hard to choose one concrete value for Ω_0 as a mean value. Curve for variable speed frame (thin line) has well defined minimum and from corresponding values of Ω_0 and B we can build a full differential rotation curve.

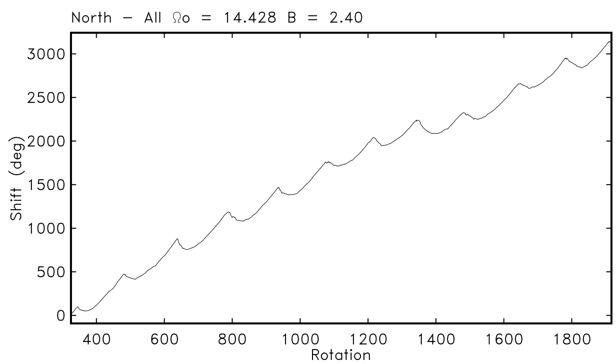

Fig. 4. The shifts for best comoving frame in Fig. 3. It is well seen that curve consists of general linearly rising part and wavy part. The minima of the wavy parts correspond to the phases of solar cycle when spots are formed at high latitudes (in average).

gitude distribution two preferred longitudes 180° apart, which migrate in any fixed rotation frame but are persistent throughout the entire period studied in certain varying speed comoving frame. From previous section we see that frames with changing rotation speed can be used to reveal short term (around, say 10 rotations) persistent longitudes with better "contrast". It is then interesesting to put two methods side by side.

– First and most important difference is that UBP method uses phase corrections in the form:

$$\Lambda_i = \Lambda_0 + T_C \sum_{j=1}^{i} (\Omega_C - \Omega_i) = = \Lambda_0 + T_C \sum_{j=1}^{i} (\Omega_C - \Omega_0 + B \sin^2 \langle \psi \rangle_j). \quad (4)$$

For positive (and physically well founded) parameter B values from this formula follows that wavy part

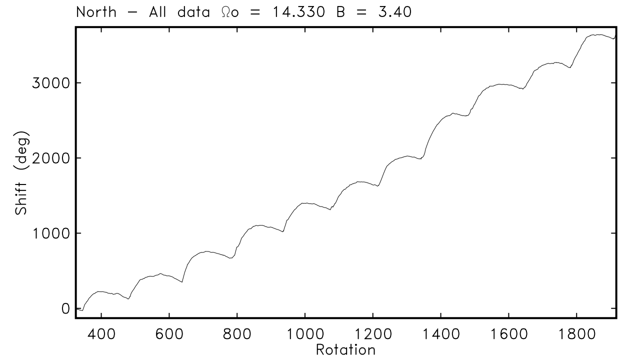

Fig. 5. The shifts for best comoving frame for Northern hemisphere. In UBP method the maxima of the wavy parts correspond to the phases of solar cycle where spots are formed at high latitudes (in average). Consequently this frame is speeded up just when spots are rotating with lesser speed.

of corrections works here in opposite to our method way. When sunspots tend to rotate slower, then these frames will speed up. This is well seen from Fig. 3-4 of BU and from Fig. 1 of UBP. And this is of course reasonable choice if the goal is to guarantee compatibility between two papers.
– The second difference is usage of sunspot areas. Our method treats only longitudes and areas are accounted for only when computing mean latitudes. In UBP areas are first normalized and then used as weights in merit function. This allows to amplify effect of these spots which lay in low activity parts of solar cycle.
– The third difference is usage of different merit function. Because in UBP authors seek a particularly shaped phase distribution they use then particularly crafted merit function.

It is not ruled out that we can have effectively two separate frames – comoving frame of the sunspot flow and frame which compensates shifts introduced by basic frame. We can imagine that there are certain density or higher intensity (due to the larger areas) waves which tend to move in opposite direction. Something similar is well known in the galaxy models.

Formally two frame parametrization models are quite similar, especially if we allow Ω_0 to vary in wide range around Ω_C and also allow negative as well positive values for parameter B. We think that our parametrization of the frame is somewhat more physical. As it was shown above it allows to recover straight forwardly differential rotation profiles and local comoving speeds.

The original paremetrization from UBP is somewhat awkward. It is built starting from certain model for differential rotation but describes essentialy quite different frame. For instance the solution for Northern hemisphere in UBP paper ($\Omega_0 = 14.33$ and $B = 3.40$) translates to similar solution in our frame - it is fixed rate rotation with slow speed on equator $2\Omega_C - \Omega_0 = 14.04$ degrees per day which cyclically speeds up (see Fig. 5). If to compare

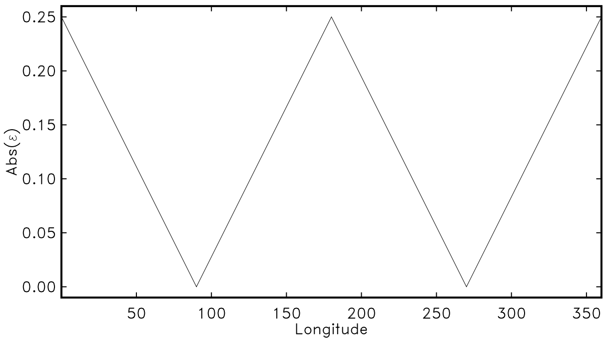

Fig. 6. Merit function depends on distance of particular phase from nearest centre (90° or 270°). It is expected that distribution seeked for will have two maxima, situated 180°-s from each other.

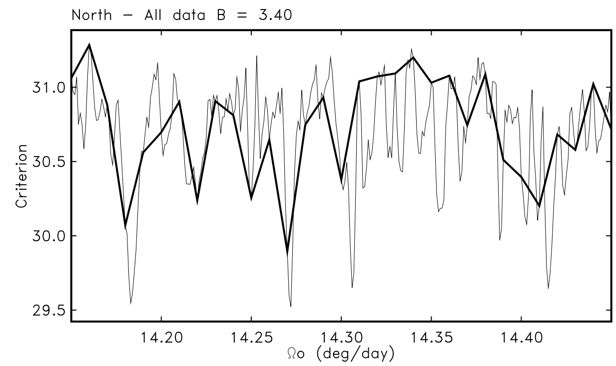

Fig. 7. Merit function dependence on Ω_0 is computed with two different time steps: 0.01 and 0.001. Insufficient precision can easily leave off significant minima.

Fig. 5 with Fig. 4 then we see that both, co-moving and contra-moving frames rotate faster than Carrington frame. The difference is in a way how the frames are composed. The physical frame in Fig. 4 is composed from positive part with slope $T_C(\Omega_0 - \Omega_C) = T_C(14.428 - 14.184)$ and negative B dependent wavy part. And *vice versa*, contra frame in Fig. 5 is composed from negative part with slope $T_C(\Omega_C - \Omega_0) = T_C(14.184 - 14.33)$ and the wavy positive part. In the first case the fast eqautorial rotation is slowed down at higher latitudes, in the second case, slow rotation at equator is speed up for higher latitudes.

Nevertheless, to be consistent and to allow simple comparisons, we will use below unphysical frame parametrization of UBP. The corresponding parameters can be easily converted to physical frame and if it is of some ineterest we will give also converted values in brackets.

Because UBP claim is much more interesting and the effect observed much less probable (if to use knowledge obtained so far) we need to be extra careful and strict in evaluation of their method and obtained results. This explains why the sections to follow are somewhat too technical.

3.1. Search for the best frame

For every combination of parameters Ω_0, B and Λ_0 we can build a correponding frame. The shifted phases are then evaluated in UBP method using merit function

$$\mathcal{E} = \frac{\sum_i \sum_k A_{ik}\epsilon_{ik}^2}{\sum_i \sum_k A_{ik}}, \tag{5}$$

where $\epsilon_{ik} = \min(\min(\tilde{\lambda}_{ik}, 360^\circ - \tilde{\lambda}_{ik}), |\tilde{\lambda}_{ik} - 180^\circ|)$ measures distance between corrected phases and nearest centre (0° or 180°) [5].

In actual computations below we used shifted in phase version of the merit function, so that expected maxima in longitude distribution were shifted to 90° and 270°. This is to ensure uniformity of the all plots. Correspondingly particular phases can differ from these in original papers. There is probably no need to say that the final results do not depend on this choice. In plots below we depict merit function values without normalization. This is just to improve readibility. In tables we present true (normalized) values of $\mathcal{E}$.

How the ϵ_{ik}-s depend on corrected phase is depicted on Fig. 6. Values in Eq. 5 A_{ik} are defined as follows

$$A_{ki} = S_{ki} / \sum_j S_{ji}, \tag{6}$$

where S is the observed area of the spot corrected for the projection effect, and the sum is taken over all spots in the given Carrington rotation.

For each particular triple $(\Lambda_0, \Omega_0\, B)$ we can compute corresponding shifts and then evaluate obtained phase (longitude) distributions using merit function $\mathcal{E}$. Before actual computations for a full parameter space it is reasonable to estimate needed step lengths along each parameter. We start from parameter Ω_0. If to look how the shifted phases depend on changes in this parameter, we see that a small change in parameter $\Delta\Omega_0$ is multiplied by constant $T_C = 25.38$ and then depending on particular rotation by number of rotations. For a last phase the number of rotations is approximately 1600.

To recover all details of the merit function it is reasonable to fix certain maximum allowed phase shift for one step change in input parameter Ω_0. Because there is two increasing and two decreasing parts in distance function (see Fig. 6) then we choose one eigth (45°) as a maximum allowed phase shift. Putting now all this together we get

$$\Delta\Omega_0 = 45/(T_C * 1600) \approx 0.001. \tag{7}$$

That this choice is rational is well demonstrated on Fig. 7 where we plotted merit function running along Ω_0 with two different steplenghts: 0.01 (thick curve, stated precision in UBP) and 0.001 (thin curve). Certainly some imortant features are lost for undersampled case. (In this

[5] In printed version of the UBP paper this function is given incorrectly. This was confirmed also by one of the authors I. Usoskin 2005

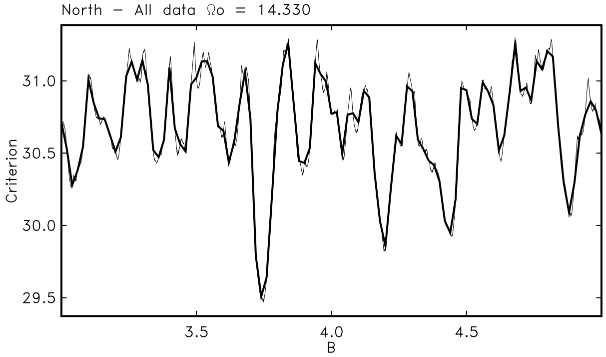

Fig. 8. Merit function depencdence on B is computed with two different time steps: 0.02 and 0.002. For the B longer step is nearly sufficient, but still some minor details can shift away from best solution.

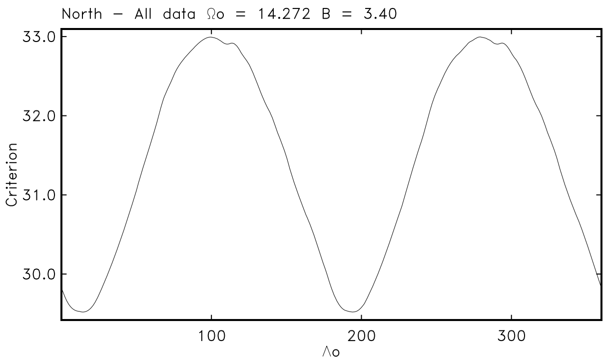

Fig. 9. The phase dependence of merit function is quite smooth. Because of symmetry of the target distribution, only half of the full phase range needs to be computed.

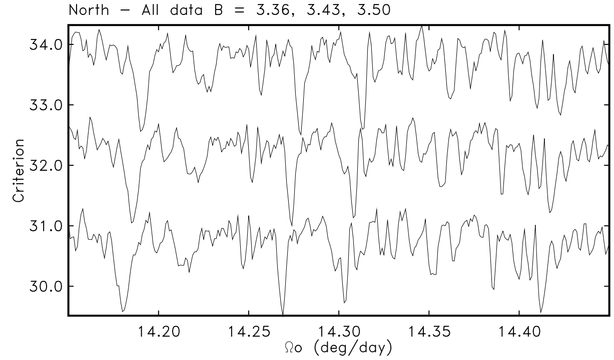

Fig. 10. Merit functions for three different values of B. It is well seen that all three are quite similar and only slightly shifted along Ω_0. On a two dimensional diagram it should look like row of slanted stripes. Nothing similar can be seen on Fig. 4 of UPB.

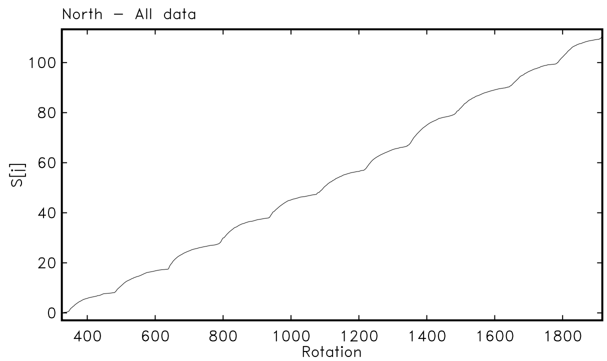

Fig. 11. Cumulative sums S_j for sequence of rotations consists of two components - linearly rising and wavelike.

plot and in all Ω_0 or B dependent plots that follow the dependency from Λ_0 is "minimized out" using steplength 5°.)

The precise analysis for the B parameter is complicated because of random component from mean latitudes. Some simple conclusions can be made by using trial and error analyis. As seen from Fig. 7 the choice of the step length 0.02 (thick curve, stated precision of the B in UBP) is not so bad, if to compare with step 0.001 (thin curve). Nevertheless, some minima can be slightly misplaced due to the undersampling. As a compromise, the reasonable choice for the B stepsize could be 0.002.

The dependence of the merit function from parameter Λ_0 is obviously quite smooth (see Fig. 5) and we can use quite rough step in degrees, say 5° to 10°. Because the distance function is periodic with period 180° we can restrict our search with subinterval $0° - 180°$.

Taking this all together we can now compute the total number of trial triples we need to work with. Say we want to seek for global merit function minimum in paparmeter space $[10 - 20] \times [0 - 6] \times [0 - 180]$ (according to one of the authors - I. Usoskin (2005), just this was the case in their initial search). Then we need evaluate at least $(10 * 1000) * (6 * 500) * 18 = 540000000$ triples. This is far beyond of our computational capabilities. For computations with significantly longer steps in parameters we can not be sure that actual global minima are recovered.

There is one possibility to significantly reduce the number of trials. Let us start from the following observation. On the Fig. 8 we depicted three merit functions for three values of parameter B. The values chosen for B are just bottom, middle and the top of values from UBP Fig. 4. It is not very complicated to see, that effectively all three functions are more or less shifted versions of the one and the same pattern.

To understand, why it is so, let us return to Eq. 1. We see that cumulated shifts depend straight forwardly from the always positive sums

$$S_j = \sum_{j=1}^{i} \sin^2\langle\psi\rangle_j, \tag{8}$$

graphically depicted in Fig. 11. This curve consists of effectively two parts: wave which models differential rotation

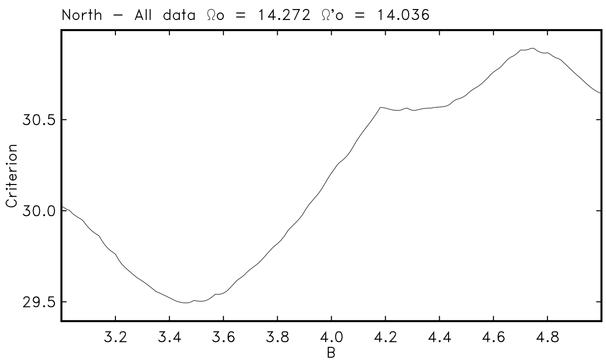

Fig. 12. For reparametrized case dependence on B is much more smoother (compare with Fig. 8).

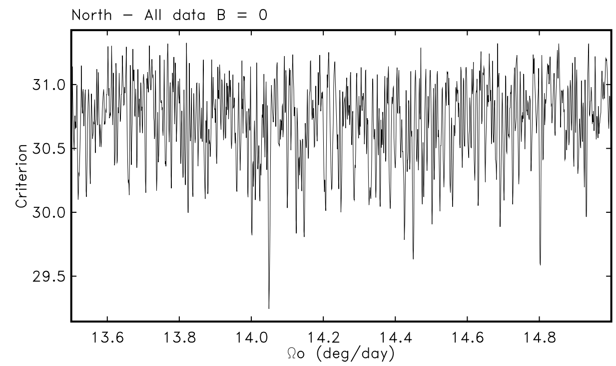

Fig. 13. Merit function computed with $B = 0$ (to seek for fixed rate frames). Northern hemisphere.

during solar cycle and linear part which is contribution to overall fixed speed differences. In final cumulative sums the fixed part is combined with similar linear component which comes from parameter Ω_0. Because these contributions are included in cumulative sums with different signs, we can compensate increase in parameter B with increase in parameter Ω_0. Their total contribution, which is difference, stays then fixed. Only thing what is changing is a result of small redistribution in phases which comes from wavy part.

To take this correlation between parameters Ω_0 and B into account we can reparametrize our analysis. First we define linear part of the curve $S_j, j = 1, \ldots, N$. Then we subtract this linear part from curve and add it to the Ω_0 dependent part. Finally we will have new parameter Ω_0' which will depend on original Ω_0 and B:

$$\Omega_0' = \Omega_0 - A_C \times B, \tag{9}$$

where $A_C = (S_N - S_1)/(N - 1)$ is linear slope for cumulative sum of S_j-s. New parameter Ω_0' will now describe overall constant speed incrementing in phases and parameter B measures how strongly we take into account corrections from differential rotation. Or if to put it another way around, the parameter Ω_0' describes now **mean comoving frame speed** and parameter B small phase shifts due to the differential rotation.

The effect of reparametrization is well seen on Fig. 12, the dependence on parameter B is much more smoother and it is possible to decrease needed number of parameter triples.

Putting all this together we can now formulate our optimization strategy. First we perform rough search using reparametrized triples. The proper steplenghths are then 0.001 for Ω_0', 0.1 for B and 5° for Λ_0. In the vicinity of the found minima we then perform refinements with steplengths 0.0001, 0.02 and 1° respectively. Standard Ω_0 can then be computed from Ω_0'.

To characterize search results and estimate their significance we will use a notion of non-axisymmetricity from UBP. For each distribution of longitudes we form two subsums of corresponding spot areas

$$N_1 = \sum_{k,i} A_{ki} \text{ , if } |\tilde{\lambda}_{ki} - 90°| < 45° \text{ or } |\tilde{\lambda}_{ki} - 270°| < 45°,$$

$$N_2 = \sum_{k,i} A_{ki} \text{ , otherwise,} \tag{10}$$

where the summation is taken over all spots in all Carrington rotations. The non-axisymmetry Γ is then defined as

$$\Gamma = \frac{N_1 - N_2}{N_1 + N_2}. \tag{11}$$

The distributions with better "contrast" will have higher values of Γ.

3.2. Search results

To minimize amount of needed computations we restricted our search space for Ω_0 to interval $[13.5 - 15.0]$. This interval includes all more or less physically plausible values for this parameter. Below we present our results in three parts. First we report our results for fixed rate frames, then for frames with particular values of B and finally for full search space. This allows as to illustrate some important aspects of merit function "surface".

3.2.1. Fixed rate frames

From previous discussion we know that it is quite easy to find particular comoving frames for separate latitude strips and even mean frames for all data together. Before using frames with B dependence it is then reasonable to check frames with $B = 0$. From UBP we learn that "...the hypothesis of rotation of the active longitudes with a fixed rate (giving $\Gamma = 0.02 - 0.03$; see Figs. 2a, 3a, and 7a,b) cannot be distinguished from the null hypothesis of the axisymmetric sunspot distribution". Our results of subset search with $B = 0$ are depicted on Fig. 13-14 and in tables 1-2.

If we compare Fig. 13 with Fig. 1 then we can see principal difference, instead of the wide depression we see

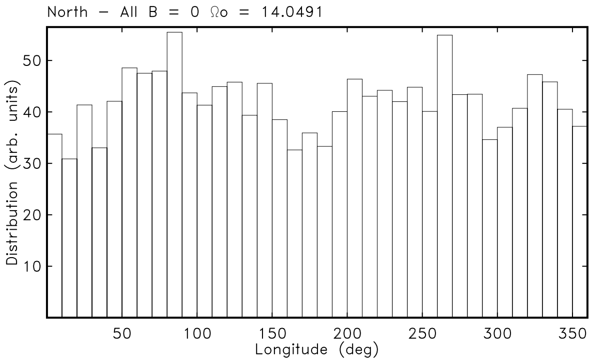

Fig. 14. Longitudinal distribution for best fixed rate comoving frame (northern hemisphere). It is somewhat fluctuating, but its non-axisymmetry $\Gamma = 0.065$ is certainly higher than $\Gamma = 0.02 - 0.03$ obtained in UBP. It is possible to amplify the impression of its wavy nature by imposing on it thick smooth model wave (as is done in UBP Fig. 2b and 3b) but we restrain ourselves in not doing so.

This work				UBP		
B	Ω_0	$\mathcal{E}$	Γ	B	Ω_0	Γ
0	14.0491	0.0194	0.065	0	?	0.02-0.03
3.40	14.272	0.0196	0.057	3.40	14.33	0.11
-3.40	14.0036	0.0195	0.064			

Table 1. Comparison of search results for particular slices for Northern hemisphere

This work				UBP		
B	Ω_0	$\mathcal{E}$	Γ	B	Ω_0	Γ
0	13.8497	0.0194	0.068	0	?	0.02-0.03
3.39	14.7324	0.0196	0.044	3.39	14.31	0.09
-3.39	13.7519	0.0192	0.073			

Table 2. Comparison of search results for particular slices for Southern hemisphere

a number of sharp minima. The strongest peaks are at ≈ 14.05 for north and ≈ 13.85 for south. Corresponding longitude distribution for Northern hemisphere is depicted in Fig. 14. The distributions are certainly not so flat as on Fig. 2,3 in UBP. We got $\Gamma = 0.065$ (North) and $\Gamma = 0.068$ (South) for non-axisymmetry, instead of $0.02 - 0,03$ in original paper.

3.2.2. Frames with particular values of B

To make straight forward comparisons with UBP results we searched for merit function minima of slices with fixed B values. For Northern hemisphere we tried to recover original result in slice with $B = 3.40$ and for Southern hemisphere with $B = 3.39$ The results are given in tables 1-2.

In addition to the computations with positive values of B we computed "spectra" also for negative values.

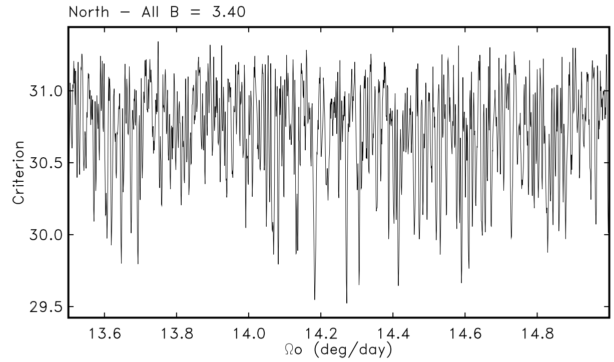

Fig. 15. Merit function computed with $B = 3.40$ for Northern hemisphere. This figure is to compared with Fig. 3 where we can see that the best comoving frame shows itself as a minimum in the middle of wide depression. Here we can see only bunch of narrow fluctuations.

It is well seen from corresponding tables that our results significantly differ from these in UBP.

Particularly interesting is Fig. 15. In the first part of the paper we saw that curve for cell counting merit function behaved quite systematically. The relevant minumum was seated in the middle of strong and wide depression (see Fig. 3). An this is quite natural. When we change parameter values, then in the vicinity of main minimum, phases migrate slowly from one cell to other and general picture changes only somewhat. What we see here, with double wave merit function, is just a fluctuating curve. There is multiple of very narrow minima scattered all over search interval.

3.2.3. Global search

Global search results are summarized in tables 3-4. Once again, there is not anything common between our results and these in UBP. May be most interesting conclusion from our results is that even when all these particular minima in merit function surface are probably just statistical fluctuations, the strongest minima tend to be for $B < 0$ - that is among comoving frames, to be contrasted with $B > 0$ contramoving frames (we still use original - unphysical parametrization here). This can be fairly easily explained. The spots occur in activity complexes and these, as was well seen in the first part of the paper, can be well described using comoving frames. Even rough double wave merit function senses (at least statistically) the general flow pattern and as a consequence tends to give "better" fluctuations just for comoving direction

As an additional sanity check, we performed also full parameter space scan for another merit function. For a moment we assumed that there are not two slowly migrating density waves but four, two for each side of the Sun. Ditribution which gave global minimum for this criterion is depicted in Fig. 17 (Northern hemisphere). It is well seen that the distribution is far from flat and con-

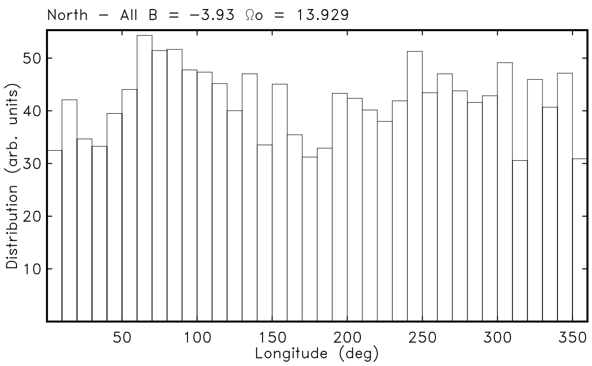

Fig. 16. Distribution which minimizes merit function in parameter space $[13.5 - 15.0] \times [-6.0 - 6.0] \times [0 - 180]$. It is not so wavy as solutions in UBP but its non-axisymmetry is quite high - $\Gamma = 0.096$.

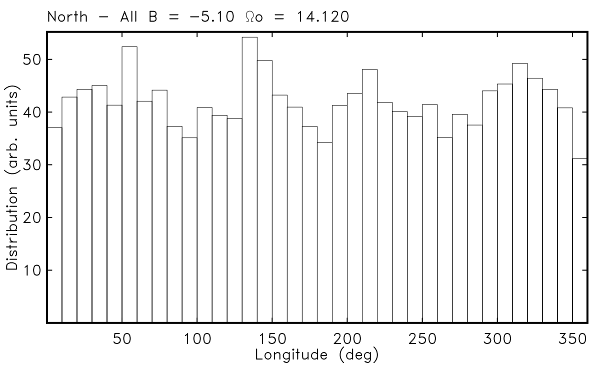

Fig. 17. When the expected distibution contains four waves (instead of two) then we get this distribution. The parameter space for search was the same as in Fig. 16.

The best distribution for Northern hemisphere is depicted in Fig. 16. The non-axisymmetricity of this distribution is $\Gamma = 0.096$ or in the terms of UBP highly significant.

This work				UBP		
B	Ω_0	$\mathcal{E}$	Γ	B	Ω_0	Γ
1.49	14.0512	0.0191	0.075	3.40	14.33	0.11
-3.93	13.929	0.0190	0.096			

Table 3. Comparison of global search results for Northern hemisphere

sequently it is possible also to develop theories with four density waves. Because of obvious reasons, we refrain from doing so.

4. Discussion

The results obtained above and these in three papers BU, UBP and BMSU are not certainly compatible. Below we try to find at least preliminary explanations for this discrepancy. We start from very general observations and end with nearly forensic type of detailed investigations. Some of our problems originate from communication difficulties

This work				UBP		
B	Ω_0	$\mathcal{E}$	Γ	B	Ω_0	Γ
0.61	13.8680	0.0191	0.073	3.39	14.31	0.09
-3.72	13.7281	0.0190	0.084			

Table 4. Comparison of global search results for Southern hemisphere

with original authors. When very open and helpful in general terms, they unfortunately had not time to answer many of our specific questions.

4.1. Where is the face of the Sun?

From the very first part of our discussion we learned that general flow of activity complexes on the Sun's surface can be esaily observed and modeled using comoving frames. The basic physical idea put forward in the three papers under scrutiny is that differential rotation hides true activity distributions which have their origin deeper inside of the Sun. To see these distributions or in metaphorical terms Sun's face, they buildt a certain contra-moving frame to roll back observed activity indicators to their proper places. As a result they got for both solar hemispheres nice bimodal distibutions. To check the significance of their results the authors performed Monte-Carlo type analysis where actual distributions were compared with purely random distributions.

Unfortunately, after careful reading of the three papers and recomputation of various statistics we are inclined to think that enigmatic face of the Sun is still hidden and that patterns what authors of three papers saw have a human made origin.

4.1.1. Oranges are not apples

The most significant problem with the persistent longitude strips is connected to their cumulative kinematics. In section 4 of the first paper (BU) we can read:

> The two active longitudes is a long-lived quasi-rigid structure, although they are not fixed in any reference frame because of the differential rotation. They continuously migrate with respect to a chosen reference frame with a variable rate. In the Carrington reference frame, the migration results in a phase lag of about 2.5 solar rotations per sunspot cycle, in total about 28 rotations for 120 years.

In follow-up study we demonstrated (see Pelt et al 2005) that at least large part of the effect can be the result of peculiar way of data processing.

From the next paper (UBP) we can read in Section 1:

> Our new analysis confirms the previous conclusions by BU03 on a new basis and dispels the doubts expressed by Pelt et al. (2005) that the active longitude separation is an artefact of the data processing.

and in section 5:

> This was criticized by Pelt et al. (2005) who claimed that the active longitude pattern found in BU03 is an artefact of the used method. The present analysis, which is based on the raw observed sunspot areas, answers their criticism and confirms that the phenomenon of the persistent active longitudes is real.

The 28 extra rotations during 120 years are not certainly ≈ 11 (North) and ≈ 14 (South). Reader can easily verify this by comparing Figs. 3-4 from BU with Fig. 1 in UBP and especially with Fig. 1 in BMSU.

Consequently the two latest papers (UBP and BMSU) **refute** results obtained in the first paper (BU).

The other kinematical problem is connected to different parametrization schemes. As we saw from the first part of our paper the physical parametrization allows to recover true flow patterns of the activity elements. But amazingly, authors use co-rotating and contra-rotating frames as equals. Probaly the best illustration to that is Fig. 6 in UBP. From figure caption we read:

> The sidereal differential rotation of the active longitudes determined in this work compared with that obtained using surface Doppler shifts ... and sunspots ...).

However in parametrization of the UBP the two frames rotate faster on higher latitudes and consequently these can not be compared straight forwardly with physical differential rotation models. The real rotation speed at equator for Northern frame in UBP is for instance 14.049 and then correspondingly faster for higher latitudes.

Third kinematical problem comes from the fact that number of extra rotations (compared with Carrington frame) is in UBP model significantly different for Northern and Southern hemispheres: ≈ 11 and ≈ 14 correspondingly (cf. Fig.1 in BMSU). If true, this means that physically we must deal with a quite significant shear component between deep and more or less rigid field components at equator plane.

4.1.2. Flip-flop revisited

Another important result of the UB is formulated so:

> The major spot activity alternates the active longitudes in about 1.53 years. In the northern and southern hemispheres this results in the alternation cycles of 3.8 and 3.65 years, respectively, which is about 1/3 of the sunspot 11-yr cycle. The difference between the cycle lengths is significant and produces the beating effect between the north and south on the century time scale. The 1/3 ratio seems to preserve on a long-time scale.

This is so called flip-flop phenomenon. The particular values for mean flip-flop periods are obtained from analysis of power spectra. In itself this result is not very convincing because authors of BU did not use correct method to estimate significance of their solution. From Fig.8-9 of the first paper reader can see that **nearly all spectral power fluctuations** are above line of 95% confidence level. But putting this aside we can now read from UBP:

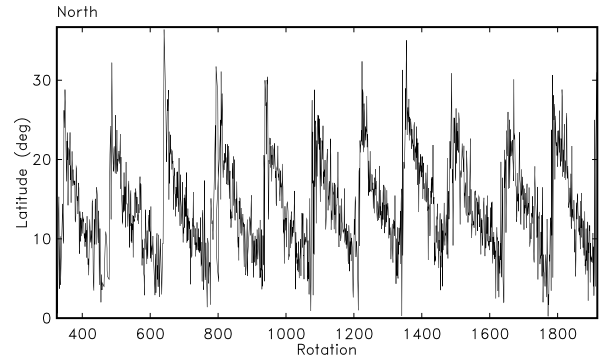

Fig. 18. Mean latitude curve is quite fluctuating. The cumulative curves formed from it (see Fig. 11 and 5) look much more smoother, but this is just a result of their monotonous nature.

> On the Sun the flip-flop phenomenon is observed as the alternation of the major spot activity between the opposite longitudes with a 3.7 year cycle (BU03).

And from BMSU we learn that:

> Of particular interest is that a flip-flop cycle of about 3.7 years was also revealed in the evolution of the spot area on the Sun (Berdyugina & Usoskin 2003).

Now, the flip-flop event frequency was computed using frame which rotates nearly three times faster than frames found in UBP and which were buildt in totally different way. Can the flip-flops remain in the same places? In principle yes. But in practice it is very hard to belive that the two phase shift systems were coherent enough. It is sufficient to say that phase shift of $\approx 180°$ which results in a move of flip-flop event position can be produced by 0.004 degrees per day change in the frame model parameter Ω_0. And shifts in a new frame model are cumulative sums of functions of quite fluctuating mean latitude curve (see Fig. 18).

4.1.3. Ground zero

The largest difference between our computations and results from UBP is behaviour of the merit function for fixed rate comoving frames ($B = 0$). From discussion above we saw that in BPU the minima of the merit function were seeked for from rather sparse grid. In principle it is then possible that some narrow peaks were left out. But as we see form Fig. 7 of the BPU the Γ values at $B = 0$ are not outliers, they are followed by values for small arguments

B at the more or less same level. Of course, we can think that behaviour of the statistic in the physically not interesting parameter range is irrelevant. But in our case it is not so.

First - high values of non-axisymmetry Γ for unphysical models show that there is high proability to get bimodal distributions which are certainly statistical fluctuations. And secondly, becacuse this part of computations uses only trivial linear longitude shift systems (instead of fluctuating frames for the case when $B > 0$), the discrepancy of the results must be result of either:

– Coding error in our software,
– Coding error in UBP software,
– Different input data sets.

It is not ruled out of course that multiple of factors is involved. Below we will describe some of our observations which we did during the debugging of our software, they were quite revealing. But here we can deal only with third possibility - different input data.

Comparing figures from BMSU and our work it is quite clear that time span of the observations and mean latitude curves are practically the same. From the UBP we learnt that authors used about 1600 Carrington rotations in their work and this is in good agreement with our 1593 rotations. Certain part of rotations contain no new activity indicators at all and some contain only zero-area spots. After removing of these rotations we were left with 1509 (North) and 1516 (South) rotations. We do not know how many rotations exactly were used in UBP but certain hints can be obtained from their Figs. 2a,2b,3a and 3b.

For instance, if we divide 1600 rotations between 36 cells then every cell will contain sum of normalized areas equal to ≈ 44 or in the case of 1509 rotations ≈ 42. Looking carefully at just mentioned figures we see that the mean occupation levels are somewhat less than these levels. We performed graphical integration of the corresponding Postscript file for Fig. 2b and got result ≈ 1330 rotations. This is (arguably of course) hint that in UBP paper certain pre- or postprocessing, automatic or manual was applied. Of course it is possible that figures are scaled "arbitrarily" (see for instance Fig. 3d were similar to the Fig. 2d distribution has two times higher occupation numbers) and then our observations are not valid.

In further remarks we assume that in UBP and then in BMSU all available rotations were used and consequently our computations used practically the same input data. If need arises, everybody can recompute our results using software and data sets in our web-site.

4.1.4. Stroboscopic effect

The one to one relationship between mean latitude curve (see Fig. 18, Fig. 5 in BMSU) and applied longitude shift curve (see Fig. 5 or Fig. 5 in BMSU) is determined by Eq. 4. For whatever reason, this relationship is called "stroboscobic effect" in BMSU. We do not fully understand the reason for that. However, the idea of the stroboscopy itself in this context brought us to the idea to look at folded (by 360°) shift curves. In Fig. 19 the shift curve for UBP contra-moving frame is depicted. The interesting feature in this plot is number of "S" form fragments. It is clear that for different frame parameters these fragments line up differently. When we build corresponding marginal distribution as in Fig. 20 we can see that the shifts themselves are not distributed evenly.

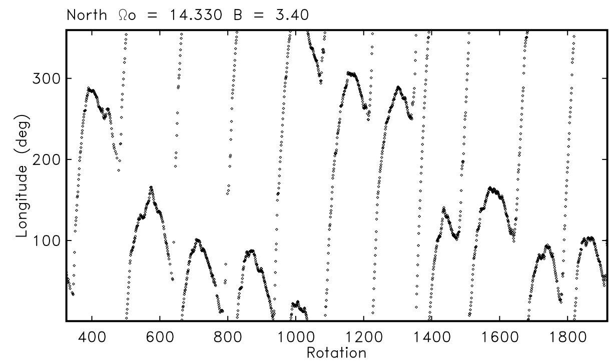

Fig. 19. Particular shifts for contramoving frame with $\Omega_0 = 13.44$ and $B = 3.40$ folded with module 360°. The "S" like features can line up to give particularly interesting marginal distributions.

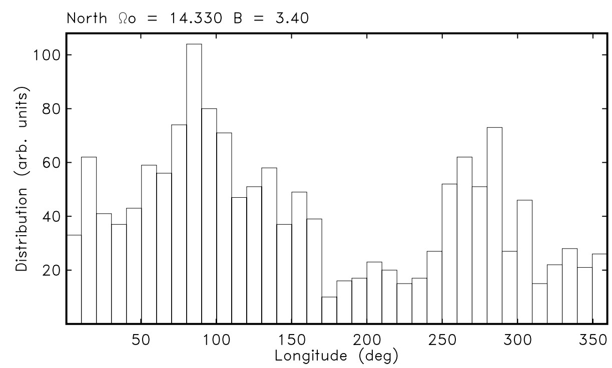

Fig. 20. Marginal distribution (along longitudes) of the frame depicted in Fig. 19.

4.1.5. Do we need longitudes at all?

From Fig. 20 we can see that for particular combination of input parameters the distribution of comoving frame shifts is far from even. Consequently it is of some interest to look how these distributions depend on parameters. In Fig. 21 it is just depicted how the merit function changes for fixed value of $B = 3.40$ and changing value Ω_0. This kind of plots can be easily buildt just by equating all longitudes exactly to zero and using then standard software. From the plot we can see that the merit function has specific wide depression and in the bottom of this depression around Ω_0 values $14.20 - 14.40$ there are quite deep minima. Let us look to this narrow region more closely in Fig. 22. As

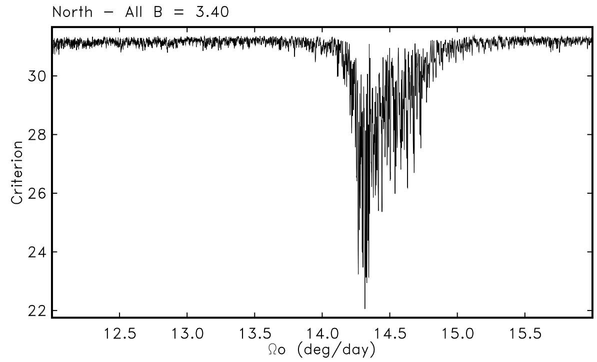

Fig. 21. Merit function which is computed with longitudes set to zero. There is strong depression around $\Omega_0 = 14.20 - 14.40$

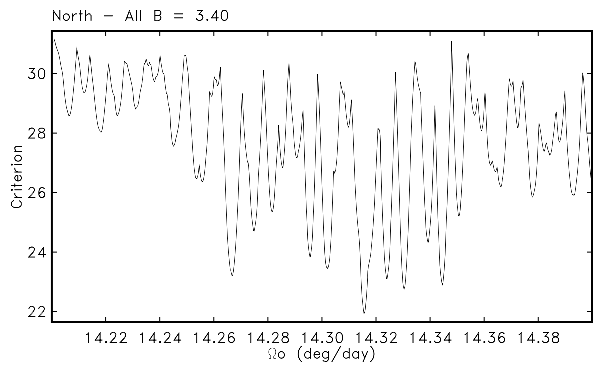

Fig. 22. Merit function at depression (see Fig. 21) reveals number of peaks. Second strongest peak is at 13.440.

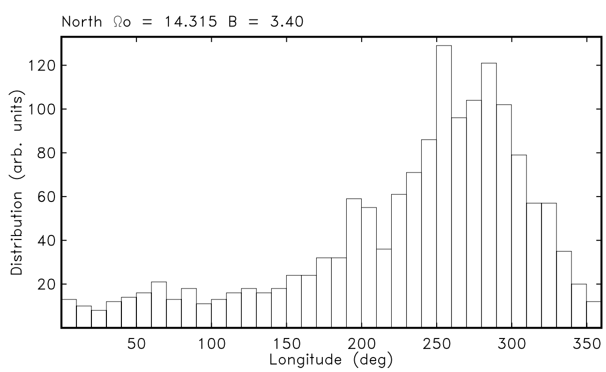

Fig. 23. Marginal distribution for frame with $\Omega_0 = 14.315$ and $B = 3.40$. It is to be compared with Fig. 20.

we can see the deepest minimum occurs at $\Omega_0 = 14.315$. The second best is at $\Omega_0 = 14.330$. The most revealing is distribution plot in Fig. 23. It occurs that value $\Omega_0 = 14.330$ is just strongest one which has characteristic double mode distribution. Just to remind you, that we **did not used longitudes at all** to compute last three plots.

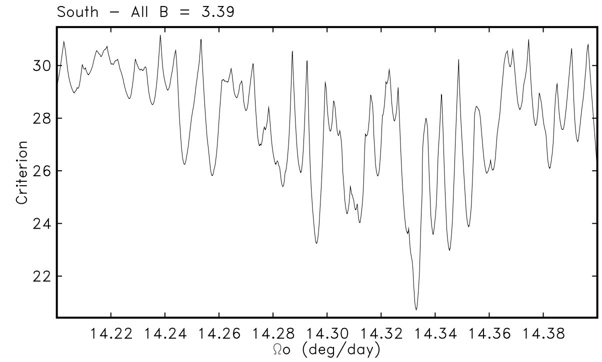

Fig. 24. Most interesting part of the merit function for Southern hemisphere. Strongest peak is at $\Omega_0 = 14.333$.

4.1.6. Can be balance between North and South re-established?

It is interesting that solution for Southern hemisphere in UBP ($B = 3.39$ and $\Omega_0 = 14.31$) has much lower "contrast" $\Gamma = 0.09$ if to compare with Northern hemisphere - $\Gamma = 0.11$. Some hints, why this is so, can be obtained from Fig. 24. The Southern global minimum for merit function (computed for frame shifts) is at 14.333. It is not ruled out that this particular fluctuation was lost in UBP analysis (it is seated far from sparse sampling points). However, it is possible also that authors saw this particular minimum, but ignored it beacuse the corresponding distribution was somewhat too asymmetric.

There is one quite mysterious plot in BPU paper, and this is their Fig. 3d. It is to compared with Fig. 2d (South with North) or with Fig. 3c (smoothed with unsmoothed). In the context of the first comparison it is not clear how the distributions differ so strongly in occupation values (from level ≈ 50 to level ≈ 100). In the context of second comparison, it is not clear how more or less symmetric distribution after smoothing becomes strongly asymmetric. Here is our hypothesis: Fig. 3d in BPU is actually leftover from experiments with universal frame model with $\Omega_0 \approx 14.33$ for Northern and Southern hemisphere together. This can explain both - occupation levels and also asymmetricity. That such an analysis was indeed performed follows from discussion in BMSU (last paragraph of Sect. 2).

We mentioned above already that the difference in number of extra rotations betwee two frames makes its physical plausibility questinable. The "solution" with 14.330 for North and 14.333 for South can pay the bill.

Unfortunately, what we are talking here about, is just certain number of fluctuations which can be found from "optimal" frame shift curves. In this and previous subsections we ignored real measured longitudes at all.

4.1.7. Secret of 14.33 revealed

But then, what peculiar number is then this 14.33 which occurs as solution in UBP and as fluctuations in input data for Northern and Southern hemispheres. The unexpected solution comes from previous analysis. To compute minima for the large parameter grid we used certain reparametrization where we divided frame shift curves into two parts: linearly rising part and wavy part. From the linear part we computed then mean rotation rate for the frame. For particular values $B = 3.40$ and $\Omega_0 = 14.330$ it gives $\Omega_0' = 14.094$. The charming magic of this number reveals itself when we compute first how much it differs from Carrington siderial rate $\Omega_C - \Omega_0' = 0.090$. From rate we can compute period and this occurs to be in years 10.96 - just quit good estimate for solar cycle length. Obtained result can be of course read out from Fig.1 in BMSU -there is approximately 11 cycles and the range of the shifts is also approximately 11 full rotations.

4.1.8. How random is random?

Previous analysis results can be refuted quite easily:

- Data sets used in UBP and here are different, they used certain pre- or postselection principles for input data (not described in a paper).
- The curves of mean latitudes (and then shift frames) in UBP were smoothed in particular way (see Fig. 1 in UBP) or were interpolated in particular way (compare Fig. 5 in BMSU and Fig. 18). We did not found any comments about these procedures in both papers.
- From fact that certain values of parameters reveal itself without taking longitudes into account does not follow that the physical effect can not still follow frames with these parameter values.
- The coincidence that particular frame found in UBP does exactly one full turn (against Carrington frame) during one solar activity cycle can well be physical effect.

But even when all these arguments are put forward there remains one particular aspect to discuss. And this is randomness itself. In UBP the significance of obtained results is evaluated using reference distribution which is computed using Monte-Carlo type methods. For each Carrington rotation they randomly permuted all the sunspots, i.e., **a new random Carrington longitudes** were ascribed to each actually observed sunspot while keeping its area. Then the value of Γ was calculated as described above.They computed the non-axisymmetry Γ for 5000 sets of such random-phase sunspot occurrence to get reference distribution. This is absolutely valid procedure for the case when we need to test certain particular longitude distributions against totally random distributions. But it is well known that on the Sun we can observe quite long-living activity complexes (see for instance Bai 1988). And from that follows that what we need to do is to start from strongly correlated activity indicator distributions and then try to evaluate how high is probability that they line up into century scale features. This is absolutely different task. And quite complicated because it needs certain estimates for strength of initial correlations.

In addition to the said correlations there is another source of flexibility in UBP method which results in quite significant fluctuations. And this comes from reweighting procedure. In input data set there are some rotations which contain nearly 70 new activity elements and all these get in method minuscule weights. But weights for the elements which happen to be on rotations between activity cycles are very powerfully amplified because these rotations are strongly underpopulated (even single new event per rotation is not very rare event). As a result it is not so complicated to get distributions with whatever we like form (see for instance Fig. 17). Instead of amplifying the effect of persitent density waves we get amplification of random fluctuations.

4.2. Is there any hope?

After two unsuccessful attempts to build apropriate comoving frames with characteristic two peaked activity distributios in BU and UBP, is there any hope at all? We think that - if only using the same input data - not. Let us explain why.

As we saw from the first part of our discussion the normal comoving flow of activity indicators can be easily seen and apropriately measured. We are certainly not talking here about fluctuations. But for density or brightness waves which travel in contra direction (at equator) the evidence can be only quite weak difference between low level real physical effect and fluctuations which result from statistical nature of the data at hand. Some authors have seen even up to the 30 year correlations in sunspot longitudinal distributions (see). It is then quite easy to combine small number of long stretches of data (using apropriate phase corrections) so that their marginal distribution will show whatever pre-crafted form we want to see.

This is of course bad luck for investigators of sun- and starspot activity. The inherent (for such an analysis) phase ambiguity is strong constraining factor which can be overcomed only by using much longer (in time) data sets. The current long time sunspot database covers only small number of correlation length size subparts and this results in high level of random fluctuations. We do not deal here with 1600 independent rotations but with significantly lower number of activity complexes.

5. Conclusions

In the recent papers (BU, UBP and BMSU) authors claim that sunspot distribution reveals that there is two persistent active longitudes which migrate according to the differential rotation law. In this paper we tried to check the results obtained. Our results can be summarized:

- The comoving frames estimated from data in BU significantly differ from frames found using mathematical optimization techniques in UBP and BMSU. In the first case the frame does approximately 28 extra rotations (compared with Carrington frame) and in the new papers the number of extra rotations is around 10-11 for Northern hemisphere and 13-14 for Southern hemisphere. Consequently the claim in UBP that new analysis confirms the results obtained in previous study is not well founded.
- In the two latest papers authors reiterate the fact of presence of the flip-flop effect with mean period 3.7 years. This result was obtained from (and then presumably incorrect) analysis in the first paper (BU).
- Our analysis demonstrated that even for fixed rate comoving frames it is quite easy to get particular longitude distributions with relatively high values of non-axisymmetry. This result is quite robust and does not depend on particular implementation of the UBP method (smoothing of the shift curve, sampling rates in parameter space etc.). Consequently the all claims in UBP about $\Gamma = 2 - 3$ level fluctuations in the case of $B = 0$ are not correct. The parameter space is full of combinations which can give rather high values for non-axisymmetry.
- Our reimplemantion of the UBP method gave absolutely different results, if to compare with results in original paper. We can only conjecture about possible reasons of discrepancies. The pre- or postselection of data points, parameter undersampling in search procedures, incorrect phase reduction etc. - these all can be considered as possible sources of differences.
- From two simple checks (Northern and Southern frames must be similar, four wave crafted model test) we conclude that frames obtained from optimization procedure are pure fluctuations.
- It occured that parameter values obtained as results in UBP can be seen already in distributions which are connected to the frame shift curve itself (which does not depend on longitudes at all).
- Simplest calculations show that the particular value for rotation parameters $\Omega_0 = 14.33, B = 3.40$ corresponds to frame which does exactly one extra rotation per solar activity cycle.

From this we conclude that the results obtained in BU, UBP and BMSU (data analysis part) are inconsistent, and evidence for well established persistent activity migration is still lacking.

In our previous paper (see Pelt 2005) we demonstrated using simple statistical models that **large part, if not all** of the persitent migration effect can be the result of data processing method used. There was certain **window of opportunity** left for more careful examination. Instead of going into the problematic details of the first model authors of UBP and BMSU preferred to build the new and different models.

The first comoving frames (with approximately 28 extra rotations in BU) revealed strong correlation with solar activity cycle. This was not statistical fluke and it was not result of the methods used. Hence it contained certain physical and relevant information. Probably it has certain sense to return to this solution instead of further analysing of the new frames were solutions obtained so far are either pure fluctuations (our work) or the results of "parasitic leakage" (UBP).

Acknowledgements. We are thankful to I.G. Usoskin for additional comments about data processing procedures used in the UBP paper. Part of this work was supported by the Estonian Science Foundation grant No. 6813.

References

Bai T., 1988, ApJ, 328, 860
Bai T., 2003, ApJ, 585, 1114
Berdyugina S.V., Moss D., Sokoloff D., Usoskin I. G., 2005, A& A preprint doi http://dx.doi.org/10.1051/20053454, (BMSU)
Berdyugina S.V., Usoskin I.G., 2003, A&A, 1121 (BU)
Jetsu L., Pelt J., Tuominen I., 1993, A&A, 278, 449
Moss D., 2004, MNRAS, 352, L17
Moss D., 2005, A&A, 432, 249
Pelt J., Brooke J., Pulkkinen P.J., Tuominen I., 2000, A& A, 362, 1143
Pelt J., Tuominen I., Brooke J., 2005, A& A, 429, 1093 (PTB)
Tuominen I., Berdyugina S.V., Korpi M.J., 2002, AN, 323, 367
Usoskin I.G., 2005, private communication
Usoskin I.G., Berdyugina S.V., Poutanen J., 2005, A& A, 441, 347 (UBP)